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An on-line Harmonics Elimination PWM Scheme for Three-Phase Voltage Source Inverter Using Quadratic Curve Fitting

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Abstract— **An on-line harmonic elimination PWM (HEPWM) scheme for three-phase voltage source inverter is proposed. It is based on quadratic curve fitting of the trajectories of the exact HEPWM angles. The main advantage of the technique is that it can be efficiently implemented using a simple low cost microprocessor. An outline to obtain the approximate HEPWM switching angles is presented. To check the accuracy of the method, an error analysis is carried out. The scheme is validated by experimental results.**

I. Introduction

The harmonics elimination PWM (HEPWM) which was originally developed by Patel and Hoft [1] offers several advantages over the conventional sinusoidal PWM (SPWM), namely:

- superior output waveform harmonic spectrum,
- higher fundamental amplitude is attainable before the minimum pulse-width limit of the inverter is reached,
- about 50% reduction in the inverter switching frequency,
- higher voltage gain due to possible over-modulation,
- smaller dc link current ripple.

Despite these numerous advantages, the implementation of HEPWM is somewhat hindered due to the fact that equations to calculate the switching angles are transcendental and therefore could not be solved on-line using digital techniques. Normally, the HEPWM switching angles are calculated off-line and subsequently stored into look up tables. However with a large number of possibilities of modulation index, ratio, and the required interpolation, the computing requirement can be quite substantial. This drawback has motivated researchers to look for methods that will allow the switching angles to be calculated on-line; thus avoiding the need for large memory storage and complicated off-line calculations. The most prominent published work was by Taufiq et.al. [2] who derived a set of non-transcendental equations for near-optimal HEPWM angles using sine-wave approximation approach. Using this scheme, the transcendental equations are "reduced" to a simpler form which permits on-line HEPWM angles computation using microprocessor. Another scheme, based on regular sampled PWM technique was suggested by Bowes [3]. Other works are mostly based on pre-calculated angles stored in memory. These are referred to as pre-programmed harmonic elimination method. Reference [4] provides a comprehensive review of this approach.

This paper proposes an alternative scheme for an on-line calculation of the HEPWM angles. It is based on the quadratic curve fitting of the trajectories of the exact HEPWM angles. The scheme is designed to eliminate the selected harmonics in a three-phase inverter system. Its main feature is the simplicity of the algorithm - only multiplications are required. It shall be shown that the proposed scheme allows for an efficient real-time computation with acceptable error margins. The viability of the scheme will be validated with hardware results.

II. The Proposed Scheme

The generalized quarter-wave symmetry HEPWM pole switching waveform with unit amplitude is shown in Fig.1. The DC voltage is "chopped" at angles α_l through α_m . The odd switching angles α_k (k odd) define the negative going transitions while the even switching angles α_k (k even) define the positive going transitions.

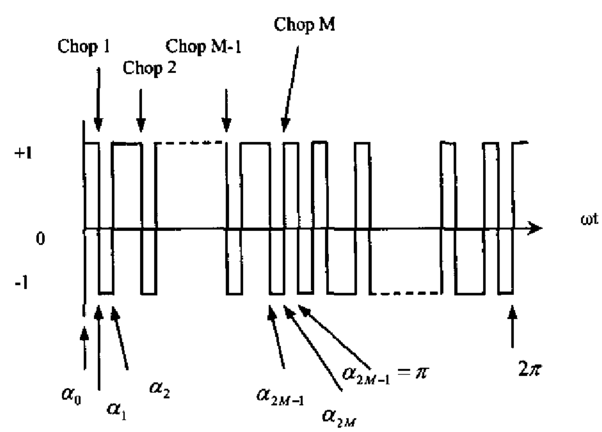


Fig.1: Generalized HEPWM pole switching waveform

The equation that describes the harmonic of this waveform is given as:

$$A_n = \frac{4}{n\pi}\left[1 + 2\sum_{k=1}^{m}(-1)^k \cos n\alpha_k\right] \quad (1)$$

and

$$B_n = 0$$

Due to quarter-wave symmetry only odd harmonics exist. Equation (1) has m variables and a set of HEPWM angles ($\alpha_1, \alpha_2\ \alpha_k$ through α_m) is obtained by equating any $m-1$ harmonics to zero and assigning a value to the fundamental component. These equations are nonlinear and transcendental and has been demonstrated to be accurately solved using numerical iteration [1].

From the solutions of (1) , trajectories for the switching angles versus the amplitude of the fundamental component of the pole switching waveform (*NP1*) can be plotted for various values of m. An example of the trajectory for m=5 is shown in Fig 2. Trajectories for other values of m will have similar pattern. From Fig. 2, for 0<*NP1*<0.8, the trajectories approximate a straight line. Hence a straight-line approximation of the trajectories can be used. However, for *NP1*> 0.8, the straight lines property diminishes and the such approximation is no longer accurate. Nevertheless, a suitable error correction factor can be introduced in this region to compensate for the non-linearity effect.

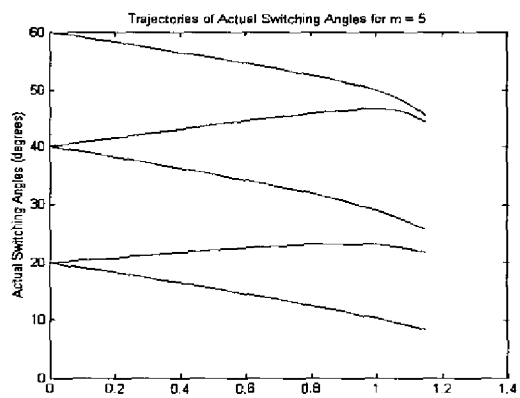


Fig.2: Switching angles trajectories for m=5

The relationship between the normalized slopes (Δ_k) of the trajectories for the different values of m can be written as:

$$\Delta_k = \frac{\dfrac{60^o(k+1)}{(m+1)} - \alpha_k}{\dfrac{2\times 60^o}{m+1}} \quad , k \text{ odd} \quad (2)$$

$$\Delta_k = \frac{\alpha_k - \dfrac{60^o(k)}{(m+1)}}{\dfrac{2\times 60^o}{m+1}} \quad , k \text{ even} \quad (3)$$

Fig.3 shows the variation of Δ_k, for odd switching angles for various values of m. The graphs suggest that the function Δ_k resembles a set of quadratic curves with nearly constant amplitude.

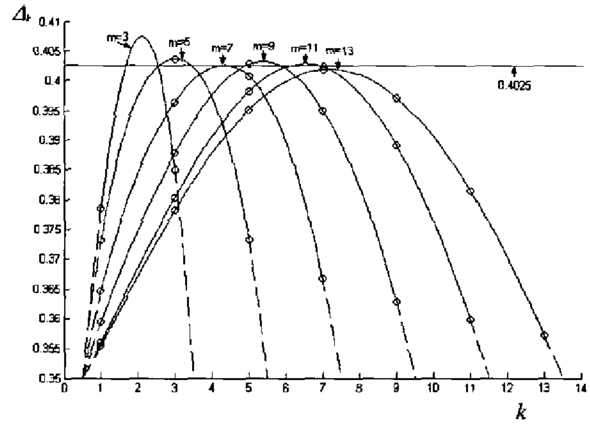


Fig. 3: Variation of Δ_k with odd k

Applying a quadratic fit to the curves, Δ_k can be expressed as:

$$\Delta_k = -\frac{0.21}{m^2}\left(k - \frac{m+1}{2}\right)^2 + 0.4025 \quad , k \text{ odd} \quad (4)$$

The generalized equation for odd switching angles, for any value of m and *NP1*, can be formulated as:

$$\alpha_k = \frac{60^o(k+1)}{m+1} - \left[\frac{2\times 60^o}{m+1} \times \frac{\Delta_k \times NP1}{0.8}\right] \quad , k \text{ odd} \quad (5)$$

For even switching angles, the variation of Δ_k with k for various values of m is shown in Fig. 4.

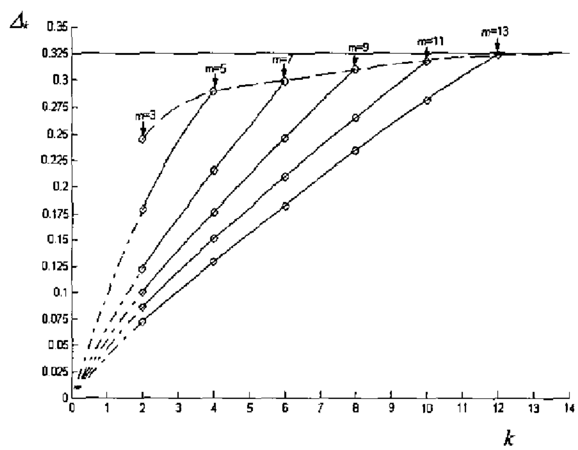


Fig. 4: Variation of Δ_k with even k

Applying curve fitting technique yields:

$$\Delta_k = -\frac{0.082}{(m-1)^2}\left[k - 2.482(m-1)\right]^2 + 0.505 - \frac{k}{m^3} \quad , k \text{ even} \qquad (6)$$

The even switching angles for any value of m and $NP1$, is then given by:

$$\alpha_k = \frac{60^o \times k}{m+1} + \left[\frac{2 \times 60^o}{m+1} \times \frac{\Delta_k \times NP1}{0.8}\right] \quad ,k \text{ even} \qquad (7)$$

Eqns. (4), (5), (6) and (7) are used to calculate the approximate HEPWM switching angles for any value of m and $NP1$. To account for non-straight line curve for the trajectories for the case for $0.8 < NP1 \leq 1.15$, an error correction scheme is incorporated. For $NP1 > 0.8$, the corrected switching angles can be formulated as:

$$\alpha_{k(corrected)} = \alpha_k - \Delta D_k \qquad (8)$$

with

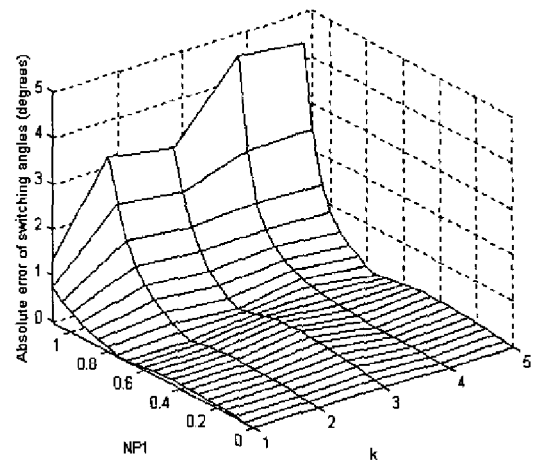
$$\Delta D_k = \frac{(NP1-0.8)^2}{0.09} \times \left[-\frac{52}{m}\left[\frac{k}{m+5} - 0.5\right]^2 + \frac{13}{m}\right] \quad \text{for odd } k \qquad (9)$$

and

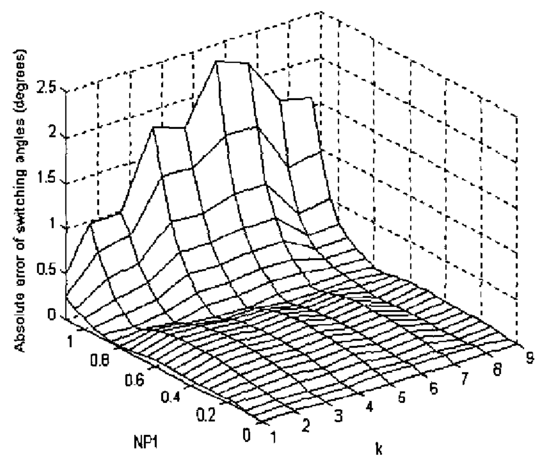
$$\Delta D_k = \frac{(NP1-0.8)^2}{0.09} \times \left[-\frac{52}{m}\left[\frac{k}{m+3} - 0.5\right]^2 + \frac{13}{m}\right] \quad \text{for even } k \qquad (10)$$

III ERROR ANALYSIS

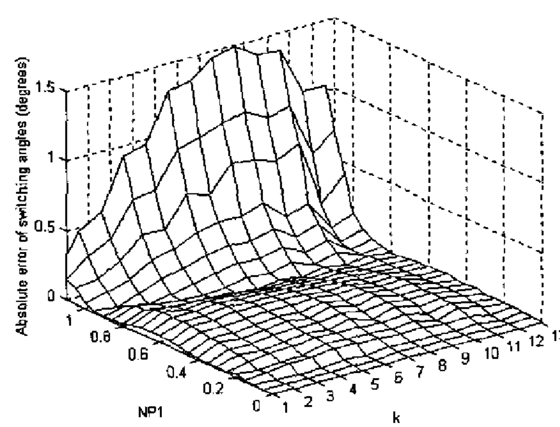
The accuracy of the derived equations is evaluated by calculating the difference between the approximate switching angles from the proposed scheme and the exact HEPWM switching angles from the trajectories. The absolute difference is termed as the angle error. The angle errors relationship with $NP1$ and the kth angle for selected values of m= 5, 9 and 13, are shown in Figures 5(a),(b) and (c), respectively. For each case, the angle error trend is very small for $NP1$ less than 0.8. In addition the errors reduce for increasing values of m. However for values of $NP1$ greater than 0.8, the errors increase drastically. The reason for this increase can be attributed to the departure of the trajectories from being straight lines for $NP1$ above 0.8. On this basis, for $NP1$ greater than 0.8, a correction factor need to be incorporated to the switching angles to reduce the error. The maximum angle errors for $0 < NP1 \leq 0.8$ are tabulated in Table 1.



(a)



(b)



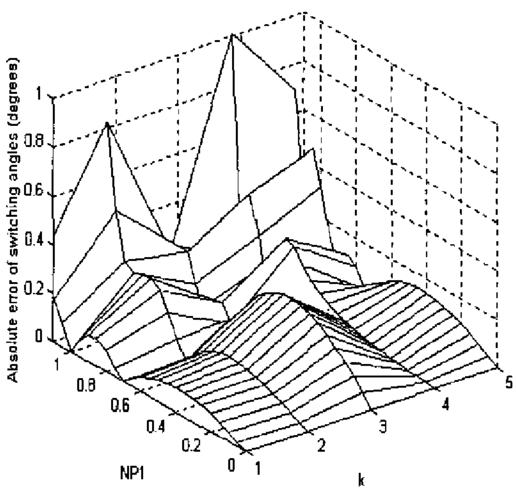
(c)

Figure 6: Variation of switching angle errors: (a) m=5, (b) m=9, (c) m=13

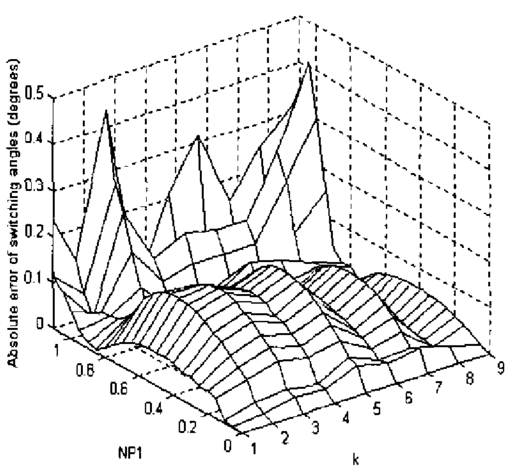
Table 1 : Maximum angle errors for $0 < NP1 \leq 0.8$

m	Odd switching angles (deg.)	Even switching angles (deg.)
3	0.6795	0.8967
5	0.3242	0.4535
7	0.2759	0.3469
9	0.2136	0.2232
11	0.1784	0.1582
13	0.1533	0.1154

Figures 7(a) through (c) show the absolute error between the exact switching angles and those calculated with the approximated quadratic equations, incorporating the correction factors. Comparing Figures 6(a) through (c), it could be seen that the maximum errors have been reduced by a factor of 3-6 times. Table 2 shows the maximum errors at $0.8 < NP1 \leq 1.15$ with and without error correction factor.



(a)



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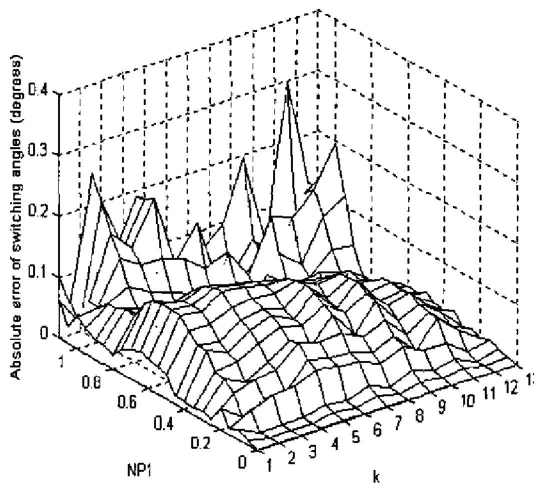

(c)

Fig. 7: Variation of angle error, with error correction factor: (a) m=5, (b) m=9, (a) m=13

Table 2 : Comparison of Maximum angle errors for $0.8 < NP1 \leq 1.15$ with Correction Factor

	Without Error Correction Factor		With Error Correction Factor	
m	Odd angles (deg.)	Even angles (deg.)	Odd angles (deg.)	Even angles (deg)
3	8.3785	8.6192	2.8490	3.3764
5	4.2015	4.3793	0.6626	0.9819
7	2.6184	2.8355	0.3697	0.6173
9	2.2420	2.1003	0.4186	0.2294
11	1.9446	1.9688	0.3606	0.4798
13	1.4446	1.4038	0.2411	0.2844

III. Implementation

To validate the algorithm, a single phase, low-power (200W) prototype inverter is built. The HEPWM waveform generation was implemented using a low-cost 16-bit Siemens 80C167 microcontroller. Since the algorithm is designed for three phase systems, the triplen harmonics will still exist in the output waveform of the single phase inverter. However, these harmonics will be cancelled when the line to line voltage is considered.Fig.5 shows the output voltage waveform of the inverter for m=5 and $NP1$=0.7. For this setting, the expected eliminated harmonics will be the 5th, 7th, 11th and 13th. Fig. 6 shows the spectra of this voltage. As can be clearly seen, the expected harmonics are successfully eliminated.

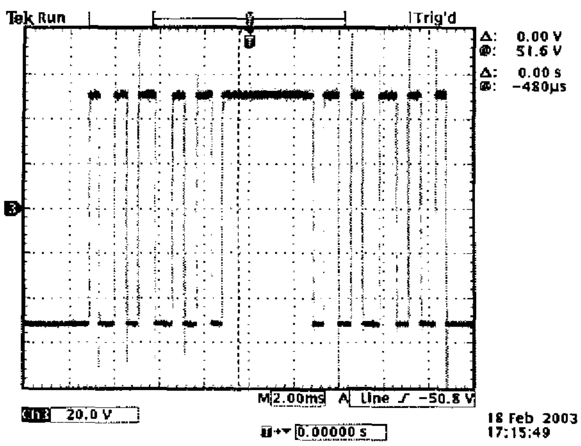

Fig. 5: Waveform of inverter output for m=5, $NP1$=0.7.

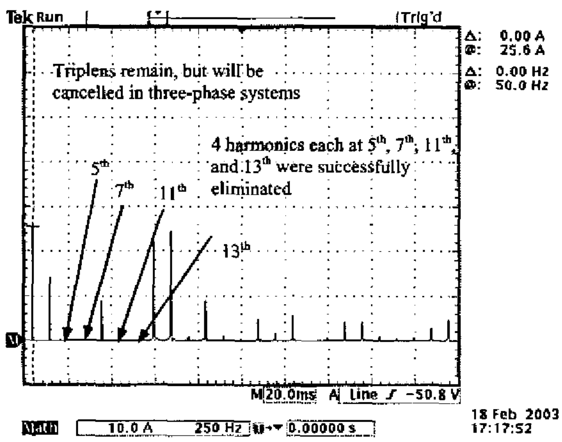

Fig. 6: Spectra of inverter output for m =5, $NP1$ =0.7.

V. Conclusion

The paper proposes a scheme to calculate the on-line switching angles using HEPWM method for a three-phase system. The algorithm results in quadratic equations, which require only the multiplication process. An outline to obtain the required HEPWM switching angles is presented. The equations are programmed using a low-cost microprocessor and implemented on a prototype inverter. It was found that the scheme is able to eliminate the intended harmonics successfully.

V. References

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